A Parametric Bootstrap Approach for Two-Way Error Component Regression Models

Lili Yue, Jianhong Shi, Weixing Song
School of Mathematics and Computer Science, Shanxi Normal University, Linfen, China
Department of Statistics, Kansas State University

Abstract In this article, the two-way error component regression model is considered. For the non-homogeneous linear hypothesis testing of regression coefficients, a parametric bootstrap (PB) approach is proposed. The proposed PB test is compared with existing generalized variable test via Monte Carlo simulation. Simulation results indicate that the PB test, regardless of the sample sizes, can control Type I error rates very satisfactorily, whereas the generalized variable test may far exceed the intended level when the sample sizes are small or moderate. Some examples to illustrate the proposed approach are presented.

Keywords parametric bootstrap; generalized variable test; simulation; two-way error component regression model.

Mathematics Subject Classification 62F03, 62F40, 62J05.

1. Introduction

There has been a continuous interest in the testing problems of two-way error component regression models. It is well known that the two-way error component regression models are special mixed models which have been widely used in econometrics, computer science, market researches and regional economic surveys, etc. However, for some problems, an exact test approach still can not be developed because that it has nuisance parameters. The generalized p-value due to Tsui and Weerahandi [9] is effective to solve the testing problems with nuisance parameters (e.g. [2,4,12,15]).

In the two-way error component regression models, testing for the regression coefficient is used in the variable selections and validity of the linearity of the models judgements. When the variance components are known, a uniformly most powerful test is available. In general, the variance components are substituted with their estimates when they are unknown. Different test statistics can be obtained from different estimating methods. If the time effects are not existed, some literatures outlined the F-test. Wu et al. [13] proposed a simple correction to the F-test variable that took into account the intracluster correlation for testing linear hypothesis. Rao et al. [6] estimated the unknown variance components and proposed a feasible F-test. Rao and Wang [7] discussed the power of these tests. Baltagi [1] suggested the one-way error component regression model can be changed into two independent linear models. Then Wang and Ma [11] proposed two exact F-tests, and they constructed a new better test by combining

Address correspondence to Jianhong Shi, School of Mathematics and Computer Science, Shanxi Normal University, Linfen 041000, China; E-mail: shijh70@163.com.

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the two F-tests. If the time effects are existed, it is not sure that the combining test can be developed. Chen et al. [2] and Fan and Wang [4] provided a generalized p-value approach for the testing problem of regression coefficients in the two-way error component regression model. Simulation results indicated, for a small number of sample sizes, the Type I error rates of the generalized variable approach may far exceed the given significant level. The PB approach has been used to provide accurate solutions when conventional methods are hard to apply or fail to perform satisfactorily; see, for example, [5,8,14]. The bootstrap methods can be carried out both parametrically and nonparametrically. However, the problems addressed in this paper are in a strict parametric setting, namely the two-way error component regression model with the usual normality assumptions. Therefore, a strict parametric bootstrap approach is developed.

The paper is organized as follows. Analysis for the two-way error component regression model is presented in Section 2. A parametric bootstrap approach for the nonhomogeneous linear hypothesis testing of regression coefficients is proposed in Section 3. The proposed test and the generalized variable approach are compared with respect to the Type I error rate and power using Monte Carlo simulation. Numerical results in Section 4 indicate that the proposed test performs better than the generalized variable test. Examples are presented in Section 5. Concluding remarks are outlined in Section 6.

2. Analysis for the Two-Way Error Component Regression Model

Consider the two-way error component regression model

$$y_{it} = \beta_0 + x'_{it}\beta + \mu_i + \nu_t + \varepsilon_{it}, \quad i = 1, 2, \cdots, N;\ t = 1, 2, \cdots, T. \tag{2.1}$$

where y_{it} is the value of the response variable at the time t on the ith individual, β_0 is an intercept and $\beta = (\beta_1, \cdots, \beta_k)'$ is an unknown vector of parameters, x_{it} denotes a $k \times 1$ vector of observations associated with y_{it}, μ_i denotes individual effect, ν_t denotes time effect and ε_{it} denotes random error vector. The error components μ_i, ν_t and ε_{it} are assumed to be independent of each other, and each in turn is independent identically distributed as $N(0, \sigma^2_\mu)$, $N(0, \sigma^2_\nu)$, $N(0, \sigma^2_\varepsilon)$, respectively, $(\sigma^2_\mu \geq 0, \sigma^2_\nu \geq 0, \sigma^2_\varepsilon > 0)$. Rewriting (2.1) in matrix form, we have

$$y = 1_{NT}\beta_0 + X\beta + u. \tag{2.2}$$

where $y = (y_{11}, \cdots, y_{1T}, y_{21}, \cdots, y_{2T}, \cdots, y_{N1}, \cdots, y_{NT})'$, $X = (x_{11}, \cdots, x_{1T}, x_{21}, \cdots, x_{2T}, \cdots, x_{N1}, \cdots, x_{NT})'$, $x_{it} = (x_{it1}, \cdots, x_{itk})'$, $u = (I_N \otimes 1_T)\mu + (1_N \otimes I_T)\nu + \varepsilon$, $\mu = (\mu_1, \cdots, \mu_N)'$, $\nu = (\nu_1, \cdots, \nu_T)'$, $\varepsilon = (\varepsilon_{11}, \cdots, \varepsilon_{1T}, \varepsilon_{21}, \cdots, \varepsilon_{2T}, \cdots, \varepsilon_{N1}, \cdots, \varepsilon_{NT})'$, and $\otimes$ denotes the Kronecker product, $rk(X) = k$. Since μ, ν, and ε are independent random variables,

$$\mathrm{Cov}(u) = \sigma^2_\mu(I_N \otimes J_T) + \sigma^2_\nu(J_N \otimes I_T) + \sigma^2_\varepsilon I_{NT}.$$

where $J_N = 1_N 1'_N$, $J_T = 1_T 1'_T$, $I_{NT} = I_N \otimes I_T$.

In this article, the following notations are used: $\bar{J}_N = J_N/N$, $\bar{J}_T = J_T/T$, $E_N = I_N - \bar{J}_N$, $E_T =$

$I_T - \bar{J}_T$. Then the covariance matrix Cov(u) can be given

$$\begin{aligned}\mathrm{Cov}(u) = & \ \sigma_\varepsilon^2(E_N \otimes E_T) + (\sigma_\varepsilon^2 + T\sigma_\mu^2)(E_N \otimes \bar{J}_T) + (\sigma_\varepsilon^2 + N\sigma_\nu^2)(\bar{J}_N \otimes E_T) \\ & +(\sigma_\varepsilon^2 + T\sigma_\mu^2 + N\sigma_\nu^2)(\bar{J}_N \otimes \bar{J}_T) \\ \triangleq & \ \sigma_1^2 Q_1 + \sigma_2^2 Q_2 + \sigma_3^2 Q_3 + \sigma_4^2 Q_4.\end{aligned} \tag{2.3}$$

where $\sigma_1^2 = \sigma_\varepsilon^2$, $\sigma_2^2 = \sigma_\varepsilon^2 + T\sigma_\mu^2$, $\sigma_3^2 = \sigma_\varepsilon^2 + N\sigma_\nu^2$, $\sigma_4^2 = \sigma_\varepsilon^2 + T\sigma_\mu^2 + N\sigma_\nu^2$, $Q_1 = E_N \otimes E_T$, $Q_2 = E_N \otimes \bar{J}_T$, $Q_3 = \bar{J}_N \otimes E_T$, $Q_4 = \bar{J}_N \otimes \bar{J}_T$. Note that, Q_1, Q_2, Q_3, Q_4 are symmetric and idempotent matrixs, they are pairwise orthogonal, $rk(Q_1) = (N-1)(T-1)$, $rk(Q_2) = N-1$, $rk(Q_3) = T-1$, $rk(Q_4) = 1$. Premultiplying (2.2) by Q_1, Q_2, Q_3, respectively

$$\begin{aligned} y_1 = Q_1 y = Q_1 X\beta + Q_1 u \triangleq X_1\beta + u_1, \\ y_2 = Q_2 y = Q_2 X\beta + Q_2 u \triangleq X_2\beta + u_2, \\ y_3 = Q_3 y = Q_3 X\beta + Q_3 u \triangleq X_3\beta + u_3. \end{aligned} \tag{2.4}$$

where $E(u_i) = 0$, Cov$(u_i) = \sigma_i^2 Q_i$, $i = 1, 2, 3$. $(X_i' X_i)^{-1}$ is assumed to be existed, this is easily satisfied in practical. From the models (2.4), some estimates can be given as follows

$$\begin{aligned} \hat{\beta}_i = (X_i'X_i)^{-1}X_i'y_i, \\ \hat{\sigma}_i^2 = \frac{\hat{u}_i'\hat{u}_i}{n_i} = \frac{y_i'(I - X_i(X_i'X_i)^{-1}X_i')y_i}{n_i}, \end{aligned} \tag{2.5}$$

where $\hat{u}_i = y_i - X_i\hat{\beta}_i$, $n_i = rk(Q_i) - rk(X) = rk(Q_i) - k$. $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$ are the best linear unbiased estimates (BLUE) of β obtained from the models (2.4) respectively by using the unified theory of least squares. $\hat{\sigma}_i^2$ is the best quadratic unbiased (BQU) estimate of σ_i^2. It is well known that $\hat{\beta}_i$ and $\hat{\sigma}_i^2$ for $i = 1, 2, 3$ are independent of each other. Furthermore,

$$V_i = \frac{n_i\hat{\sigma}_i^2}{\sigma_i^2} \sim \chi^2_{n_i}, \quad i = 1, 2, 3.$$

For the model (2.2), the best linear unbiased estimate of β is

$$\hat{\beta}_{GLS} = (\frac{X_1'X_1}{\sigma_1^2} + \frac{X_2'X_2}{\sigma_2^2} + \frac{X_3'X_3}{\sigma_3^2})^{-1}(\frac{X_1'y_1}{\sigma_1^2} + \frac{X_2'y_2}{\sigma_2^2} + \frac{X_3'y_3}{\sigma_3^2}), \tag{2.6}$$

but, it is not a feasible estimate because variance components may be unknown. In this case, a two-step estimate can be obtained by replacing σ_i^2 in (2.6) with $\hat{\sigma}_i^2$, it is given by

$$\hat{\beta}_{TS} = (\frac{X_1'X_1}{\hat{\sigma}_1^2} + \frac{X_2'X_2}{\hat{\sigma}_2^2} + \frac{X_3'X_3}{\hat{\sigma}_3^2})^{-1}(\frac{X_1'y_1}{\hat{\sigma}_1^2} + \frac{X_2'y_2}{\hat{\sigma}_2^2} + \frac{X_3'y_3}{\hat{\sigma}_3^2}). \tag{2.7}$$

By now, the two-step estimate $\hat{\beta}_{TS}$ is feasible and it has its optimalities. The fact has been studied in some literatures. When $\sigma_\nu^2 = 0$, Wang and Fan [10] discussed the optimality of $\hat{\beta}_{TS}$. It is well known that OLS estimate is unbiased, but asymptotically inefficient. True GLS estimate is BLUE, but the variance components are usually unknown and have to be estimated. All the feasible GLS estimates considered are asymptotically efficient, and the fact of $\hat{\beta}_{TS}$ and $\hat{\sigma}_i^2$ are mutually independent can be carried out. In view of these facts, the test variable will be developed using $\hat{\beta}_{TS}$. In this paper, we denote $B_1 = X_1'X_1$, $B_2 = X_2'X_2$, $B_3 = X_3'X_3$. Then,

$$\hat{\beta}_{TS}|\hat{\sigma}^2 \sim N(\beta, V),$$

where $\hat{\sigma}^2 = (\hat{\sigma}_1^2, \hat{\sigma}_2^2, \hat{\sigma}_3^2)'$, $V = (\frac{1}{\hat{\sigma}_1^2}B_1 + \frac{1}{\hat{\sigma}_2^2}B_2 + \frac{1}{\hat{\sigma}_3^2}B_3)^{-1}(\frac{\sigma_1^2}{\hat{\sigma}_1^4}B_1 + \frac{\sigma_2^2}{\hat{\sigma}_2^4}B_2 + \frac{\sigma_3^2}{\hat{\sigma}_3^4}B_3)(\frac{1}{\hat{\sigma}_1^2}B_1 + \frac{1}{\hat{\sigma}_2^2}B_2 + \frac{1}{\hat{\sigma}_3^2}B_3)^{-1}$.

3. The Parametric Bootstrap Approach

In this section, a parametric bootstrap approach for the testing problem of regression coefficients in the two-way error component regression model is developed. The following hypothesis is interested

$$H_0 : H\beta = d \leftrightarrow H_1 : H\beta \neq d \tag{3.1}$$

where H is a $p \times k$ matrix with dimension of p, d is a $p \times 1$ known vector. Define

$$T(\sigma_1^2, \sigma_2^2, \sigma_3^2) = (H\hat{\beta}_{TS} - d)'(HVH')^{-1}(H\hat{\beta}_{TS} - d), \tag{3.2}$$

If σ_i^2's are known, $T(\sigma_1^2, \sigma_2^2, \sigma_3^2)$ can be a statistic for testing (3.1). In fact, when the null hypothesis H_0 in (3.1) is satisfied, $(HVH')^{-1/2}(H\hat{\beta}_{TS} - d) \sim N(0, I_p)$, so

$$(H\hat{\beta}_{TS} - d)'(HVH')^{-1}(H\hat{\beta}_{TS} - d) \sim \chi_p^2.$$

The test that rejects H_0 at nominal level α whenever

$$T(\sigma_1^2, \sigma_2^2, \sigma_3^2) > \chi_{p,\alpha}^2.$$

In fact, the variances components σ_i^2's are often unknown, in this case, a test statistic can be developed by replacing σ_i^2 with $\hat{\sigma}_i^2$ in (3.2), $i = 1, 2, 3$, and is expressed as

$$T(\hat{\sigma}_1^2, \hat{\sigma}_2^2, \hat{\sigma}_3^2) = (H\hat{\beta}_{TS} - d)'(H(\frac{B_1}{\hat{\sigma}_1^2} + \frac{B_2}{\hat{\sigma}_2^2} + \frac{B_3}{\hat{\sigma}_3^2})^{-1}H')^{-1}(H\hat{\beta}_{TS} - d). \tag{3.3}$$

The parametric bootstrap approach involves sampling from the estimated models. That is, samples or sample statistics are generated from parametric models with the parameters substituted by their estimates. Under null hypothesis $H_0 : H\beta - d = 0$ in (3.1), the PB pivot variable based on the test statistic $T(\hat{\sigma}_1^2, \hat{\sigma}_2^2, \hat{\sigma}_3^2)$ in (3.3) can be developed as

$$TB(S_{B1}^2, S_{B2}^2, S_{B3}^2) = (H\hat{\beta}_{TS} - d)'(H(\frac{B_1}{S_{B1}^2} + \frac{B_2}{S_{B2}^2} + \frac{B_3}{S_{B3}^2})^{-1}H')^{-1}(H\hat{\beta}_{TS} - d), \tag{3.4}$$

with

$$(H\hat{\beta}_{TS} - d) \sim N(0, H(\frac{B_1}{\hat{\sigma}_1^2} + \frac{B_2}{\hat{\sigma}_2^2} + \frac{B_3}{\hat{\sigma}_3^2})^{-1}H'), \tag{3.5}$$

$$S_{Bi}^2 \sim \frac{\hat{\sigma}_i^2}{n_i}\chi_{n_i}^2, \quad i = 1, 2, 3. \tag{3.6}$$

the PB test rejects H_0 in (3.1) when

$$P(TB(S_{B1}^2, S_{B2}^2, S_{B3}^2) > t) < \alpha, \tag{3.7}$$

where

$$t = T(s_1^2, s_2^2, s_3^2)$$

is an observed value of $T(\hat{\sigma}_1^2, \hat{\sigma}_2^2, \hat{\sigma}_3^2)$ in (3.3), α is a given nominal level. It is not difficult to note that the above probability does not rely on any unknown parameters, and hence it can be evaluated by Monte Carlo simulation given in Algorithm 1.

Algorithm 1. For a given s_1^2, s_2^2, s_3^2 :
compute $T(s_1^2, s_2^2, s_3^2)$ in (3.3) and call it t
For $k = 1, 2, \cdots, m$
generate $(H\hat{\beta}_{TS} - d) \sim N(0, H(\frac{B_1}{\hat{\sigma}_1^2} + \frac{B_2}{\hat{\sigma}_2^2} + \frac{B_3}{\hat{\sigma}_3^2})^{-1}H'),\ S_{Bi}^2 \sim \frac{\hat{\sigma}_i^2}{n_i}\chi_{n_i}^2,\quad i = 1, 2, 3$
compute $TB(S_{B1}^2, S_{B2}^2, S_{B3}^2)$ using (3.4)
if $TB(S_{B1}^2, S_{B2}^2, S_{B3}^2) > t$, set $Q_k = 1$
(end loop)
$(1/m)\sum_{k=1}^{m} Q_k$ is an estimate of p-value in (3.7) by using Monte Carlo simulation.

In the remainder of this section, we present the generalized variable (GV) test developed by Fan and Wang (2008) in comparison to that of the proposed approach. The method developed in [4] will be briefly review as follows. The generalized test variable for the hypothesis in (3.1) is expressed as

$$\begin{aligned} T_1 = &\ [(HVH')^{-1/2}(H\hat{\beta}_{TS} - d)]'H(\tfrac{1}{s_1^2}B_1 + \tfrac{1}{s_2^2}B_2 + \tfrac{1}{s_3^2}B_3)^{-1}(\tfrac{\sigma_1^2}{s_1^2\hat{\sigma}_1^2}B_1 + \tfrac{\sigma_2^2}{s_2^2\hat{\sigma}_2^2}B_2 + \tfrac{\sigma_3^2}{s_3^2\hat{\sigma}_3^2}B_3) \\ &(\tfrac{1}{s_1^2}B_1 + \tfrac{1}{s_2^2}B_2 + \tfrac{1}{s_3^2}B_3)^{-1}H'[(HVH')^{-1/2}(H\hat{\beta}_{TS} - d)] \\ \triangleq &\ \xi'H(\tfrac{B_1}{s_1^2} + \tfrac{B_2}{s_2^2} + \tfrac{B_3}{s_3^2})^{-1}(\tfrac{B_1}{s_1^2\eta_1} + \tfrac{B_2}{s_2^2\eta_2} + \tfrac{B_3}{s_3^2\eta_3})(\tfrac{B_1}{s_1^2} + \tfrac{B_2}{s_2^2} + \tfrac{B_3}{s_3^2})^{-1}H'\xi, \end{aligned} \tag{3.8}$$

where s_i^2 is the observed value of $\hat{\sigma}_i^2,\ i = 1, 2, 3$, when H_0 in (3.1) is satisfied, $\xi = (HVH')^{-1/2}(H\hat{\beta}_{TS} - d) \sim N(0, I_p)$, $\eta_i = \hat{\sigma}_i^2/\sigma_i^2 \sim \chi_{n_i}^2/n_i$, and ξ, η_i are mutually independent. $t_1 = (H\hat{\beta}_{TS} - d)'(H\hat{\beta}_{TS} - d)$ is the observed value of T_1 and it is free of unknown parameters, the distribution of T_1 is free of the nuisance parameters when $H\beta = d$ is contented. In addition, T_1 is nondecreasing in $(H\beta - d)'(H\beta - d)$. Therefore, the generalized p-value can be obtained as

$$p = P(T_1 > t_1|H_0). \tag{3.9}$$

The generalized variable approach rejects the hypothesis H_0 in (3.1) whenever the probability in (3.9) is less than a given significant level α. Note that, the probability in (3.9) doesn't rely on any unknown parameters, hence it can be evaluated using Monte Carlo simulation.

4. Numerical results

To assess the performance of the proposed PB approach, some simulation studies are conducted in this section. In order to evaluate the Type I error rates of the PB and GV tests, two-step simulations are needed. The Monte Carlo simulation method applied to estimate the Type I error rates of the PB approach is expressed as follows. When the sample size is given, we used 2500 runs for outer 'do loop' (for generating the observed vectors, $(H\hat{\beta}_{TS} - d)$, (s_1^2, s_2^2, s_3^2), and then the observed value t of $T(\hat{\sigma}_1^2, \hat{\sigma}_2^2, \hat{\sigma}_3^2)$ in (3.3) was computed), under $H_0 : H\beta - d = 0$, $H\hat{\beta}_{TS} - d \sim N(0, HVH')$, $s_i^2 \sim \sigma_i^2\chi_{n_i}^2/n_i,\ i = 1, 2, 3$, they are generated independently. For each of the computed t's, 5000 runs for 'inner loop' to generate bootstrap samples and then estimate the p-value in (3.7) are used. Finally, the Type I error rate of the PB test was evaluated by the proportion of the 2500 p-values that are less than the given significant level α. The Type I error rates of the GV test was estimated by the similar method. For the powers of the

PB and GV tests, a similar simulation method can be applied except for taking different b values. The computational procedure was realized in the Matlab circumstance.

In our simulations, X is a design matrix which can be randomly generated from a normal distribution, the nominal level is given as $\alpha = 0.05$, $b = (H\beta - d)'(H\beta - d)$. The parameter settings of the simulation studies are as follows: (1) The sample sizes rang from small, moderate to large: the number of individual effects equals 4, 6, 8, 10, 13, 15, the number of time effects is 5, 8, 10; (2) The dimensions of the regression coefficients are $k = 2, 3, 4$, $H = (1, 1)$, $\beta = (1, 2)'$, $H = (1, 1, 1)$, $\beta = (1, 2, 1)'$ and $H = (1, 1, 1, 1)$, $\beta = (1, 2, 1, 2)'$ are assumed respectively; (3) We consider the scenarios with $\sigma_\varepsilon^2 = 1$, $\sigma_\mu^2 + \sigma_\nu^2 = 12$, or $\sigma_\mu^2 + \sigma_\nu^2 = 22$. The data with underlining are the Type I error rates of both tests.

Table 1

Simulation of Type I error rates and powers when $N = 4, 6, 8$; $T = 5$; $k = 2$; $\sigma_\varepsilon^2 = 1$; $\sigma_\mu^2 + \sigma_\nu^2 = 12$.

$N = 4, T = 5, k = 2$									
σ_μ^2	Tests	$b = 0$	$b = 0.01$	$b = 4$	$b = 6.76$	$b = 10.89$	$b = 11.56$	$b = 12.96$	$b = 13.69$
4	PB	0.0564	0.0600	0.7940	0.8932	0.9476	0.9540	0.9628	0.9652
	GV	0.0772	0.0804	0.7488	0.8544	0.9148	0.9208	0.9320	0.9364
6	PB	0.0576	0.0612	0.8096	0.9068	0.9540	0.9596	0.9680	0.9708
	GV	0.0800	0.0832	0.7676	0.8768	0.9244	0.9304	0.9420	0.9456
8	PB	0.0596	0.0612	0.8244	0.9128	0.9580	0.9644	0.9696	0.9724
	GV	0.0816	0.0848	0.7816	0.8836	0.9352	0.9392	0.9452	0.9480
$N = 6, T = 5, k = 2$									
σ_μ^2	Tests	$b = 0$	$b = 0.04$	$b = 0.25$	$b = 1$	$b = 1.21$	$b = 1.44$	$b = 1.69$	$b = 1.96$
4	PB	0.0452	0.1060	0.3268	0.7856	0.8468	0.8860	0.9200	0.9464
	GV	0.0756	0.1044	0.3016	0.7788	0.8256	0.8764	0.9100	0.9428
6	PB	0.0512	0.1060	0.3260	0.8076	0.8452	0.8856	0.9204	0.9428
	GV	0.0748	0.1028	0.3016	0.7760	0.8248	0.8712	0.9172	0.9356
8	PB	0.0524	0.1056	0.3236	0.7816	0.8392	0.8836	0.9204	0.9420
	GV	0.0768	0.1040	0.2996	0.7692	0.8392	0.8716	0.9148	0.9360
$N = 8, T = 5, k = 2$									
σ_μ^2	Tests	$b = 0$	$b = 0.09$	$b = 0.49$	$b = 0.64$	$b = 0.81$	$b = 1$	$b = 1.21$	$b = 1.44$
4	PB	0.0532	0.1960	0.6972	0.7948	0.8736	0.9116	0.9504	0.9688
	GV	0.0684	0.1916	0.6908	0.7908	0.8640	0.9092	0.9472	0.9688
6	PB	0.0536	0.1944	0.6944	0.7876	0.8708	0.9088	0.9484	0.9676
	GV	0.0676	0.1920	0.6848	0.7852	0.8596	0.9080	0.9456	0.9656
8	PB	0.0528	0.1940	0.6924	0.7844	0.8708	0.9176	0.9464	0.9648
	GV	0.0688	0.1884	0.6804	0.7804	0.8576	0.9064	0.9440	0.9644

Table 2

Simulation of Type I error rates and powers when $N = 10$; $T = 5$; $k = 2$; $\sigma_\varepsilon^2 = 1$; $\sigma_\mu^2 + \sigma_\nu^2 = 12, 22$.

$\sigma_\mu^2 + \sigma_\nu^2 = 12$									
σ_μ^2	Tests	$b = 0$	$b = 0.04$	$b = 0.16$	$b = 0.25$	$b = 0.36$	$b = 0.49$	$b = 0.64$	$b = 0.81$
4	PB	0.0492	0.1276	0.3700	0.5300	0.6720	0.7980	0.8756	0.9280
	GV	0.0604	0.1412	0.3720	0.5260	0.6636	0.7856	0.8756	0.9228
6	PB	0.0480	0.1272	0.3676	0.5284	0.6728	0.7968	0.8732	0.9264
	GV	0.0620	0.1452	0.3680	0.5236	0.6600	0.7900	0.8724	0.9224
8	PB	0.0508	0.1276	0.3652	0.5248	0.6684	0.7928	0.8708	0.9252
	GV	0.0644	0.1464	0.3636	0.5212	0.6568	0.7856	0.8692	0.9208
$\sigma_\mu^2 + \sigma_\nu^2 = 22$									
σ_μ^2	Tests	$b = 0$	$b = 0.04$	$b = 0.16$	$b = 0.25$	$b = 0.36$	$b = 0.49$	$b = 0.64$	$b = 0.81$
4	PB	0.0496	0.1292	0.3712	0.5308	0.6744	0.8008	0.8740	0.9400
	GV	0.0560	0.1428	0.3732	0.5276	0.6684	0.7964	0.8640	0.9340
6	PB	0.0476	0.1280	0.3688	0.5296	0.6728	0.7984	0.8724	0.9388
	GV	0.0596	0.1436	0.3716	0.5240	0.6648	0.7776	0.8632	0.9324
8	PB	0.0476	0.1276	0.3680	0.5300	0.6720	0.7980	0.8864	0.9276
	GV	0.0588	0.1452	0.3688	0.5232	0.6656	0.7928	0.8712	0.9272
12	PB	0.0468	0.1276	0.3668	0.5284	0.6712	0.7964	0.8720	0.9268
	GV	0.0580	0.1452	0.3684	0.5224	0.6624	0.7884	0.8712	0.9240
16	PB	0.0464	0.1272	0.3652	0.5264	0.6704	0.7932	0.8700	0.9272
	GV	0.0584	0.1456	0.3656	0.5204	0.6580	0.7836	0.8684	0.9196
18	PB	0.0472	0.1276	0.3640	0.5228	0.6676	0.7896	0.8684	0.9348
	GV	0.0616	0.1456	0.3636	0.5200	0.6552	0.7800	0.8676	0.9216

Table 3

Simulation of Type I error rates and powers when $N = 13$; $T = 8$; $k = 2, 3$; $\sigma_\varepsilon^2 = 1$; $\sigma_\mu^2 + \sigma_\nu^2 = 12, 22$.

$k = 2$									
σ_μ^2	Tests	$b = 0$	$b = 0.01$	$b = 0.04$	$b = 0.09$	$b = 0.16$	$b = 0.25$	$b = 0.36$	$b = 0.64$
4	PB	0.0520	0.1040	0.2452	0.4924	0.7272	0.8816	0.9600	0.9984
	GV	0.0516	0.0904	0.2436	0.4856	0.7200	0.8776	0.9596	0.9972
6	PB	0.0520	0.1040	0.2452	0.4916	0.7252	0.8816	0.9628	0.9980
	GV	0.0516	0.0904	0.2428	0.4840	0.7192	0.8772	0.9544	0.9972
8	PB	0.0524	0.1040	0.2636	0.4912	0.7256	0.8816	0.9628	0.9980
	GV	0.0512	0.0896	0.2420	0.4824	0.7204	0.8784	0.9548	0.9972

(Continued)

Table 3

(Continued)

				$k=3$					
σ_μ^2	Tests	$b=0$	$b=0.01$	$b=0.04$	$b=0.09$	$b=0.16$	$b=0.25$	$b=0.36$	$b=0.64$
4	PB	0.0488	0.0852	0.1904	0.3544	0.5508	0.7432	0.8776	0.9836
	GV	0.0516	0.0832	0.1848	0.3512	0.5392	0.7252	0.8736	0.9800
6	PB	0.0476	0.0852	0.1892	0.3544	0.5504	0.7424	0.8764	0.9832
	GV	0.0428	0.0836	0.1844	0.3492	0.5380	0.7252	0.8744	0.9796
8	PB	0.0488	0.0852	0.1892	0.3544	0.5508	0.7424	0.8752	0.9832
	GV	0.0516	0.0836	0.1844	0.3480	0.5392	0.7260	0.8736	0.9800
12	PB	0.0488	0.0848	0.1892	0.3544	0.5504	0.7416	0.8752	0.9832
	GV	0.0532	0.0844	0.1872	0.3476	0.5400	0.7276	0.8728	0.9788
16	PB	0.0492	0.0848	0.1892	0.3544	0.5504	0.7416	0.8752	0.9832
	GV	0.0536	0.0844	0.1880	0.3472	0.5380	0.7256	0.8736	0.9780
18	PB	0.0476	0.0852	0.1892	0.3540	0.5508	0.7424	0.8756	0.9832
	GV	0.0444	0.0844	0.1860	0.3460	0.5392	0.7244	0.8728	0.9784

Table 4

Simulation of Type I error rates and powers when $N=15$; $T=10$; $k=4$; $\sigma_\varepsilon^2=1$; $\sigma_\mu^2+\sigma_\nu^2=22$.

				$N=15, T=10, k=4$					
σ_μ^2	Tests	$b=0$	$b=0.01$	$b=0.04$	$b=0.09$	$b=0.16$	$b=0.25$	$b=0.36$	$b=0.49$
4	PB	0.0488	0.0904	0.2188	0.3932	0.6104	0.7860	0.9108	0.9696
	GV	0.0552	0.0832	0.1888	0.3724	0.5860	0.7764	0.8908	0.9648
6	PB	0.0488	0.0904	0.2184	0.3928	0.6088	0.7856	0.9108	0.9696
	GV	0.0540	0.0816	0.1876	0.3708	0.5852	0.7748	0.8904	0.9648
8	PB	0.0488	0.0904	0.2184	0.3928	0.6088	0.7856	0.9108	0.9696
	GV	0.0544	0.0812	0.1860	0.3700	0.5832	0.7748	0.8896	0.9640
12	PB	0.0488	0.0904	0.2180	0.3928	0.6088	0.7852	0.9108	0.9688
	GV	0.0540	0.0808	0.1864	0.3700	0.5832	0.7728	0.9092	0.9644
16	PB	0.0492	0.0904	0.2180	0.3924	0.6088	0.7856	0.9104	0.9688
	GV	0.0536	0.0812	0.1868	0.3700	0.5832	0.7720	0.9088	0.9648
18	PB	0.0492	0.0904	0.2184	0.3928	0.6088	0.7856	0.9104	0.9692
	GV	0.0524	0.0824	0.1872	0.3704	0.5832	0.7716	0.9080	0.9644

Tables 1-4 present the estimated Type I error rates and powers of the PB and GV tests. Simulation results clearly indicate that the PB test outperforms the generalized variable test in terms of maintaining the intended level and power. The powers of both tests are monotonically increasing functions of b. We observe the following from the numerical results in Tables 1 and 2: (1) For small and moderate sample, the PB test controls the Type I error rates very close to the nominal level, whereas the GV test may far exceed the intended level. In the worst cases, the Type I error rates of the PB test are around 0.0576 whereas the Type I error rates of the GV test goes as high as 0.0816 when the nominal level is 0.05. That is, the Type I errors of the PB test may exceed the given significant level considerably but not to the extent of the GV test; (2) The power of the proposed test is a little higher than Fan's GV test; (3) For the case of $\sigma_\mu^2 + \sigma_\nu^2 = 12$, the tests exhibit similar performance as in the case of $\sigma_\mu^2 + \sigma_\nu^2 = 22$. The results in Table 3-4 indicate that for large sample sizes, both tests control Type I error rates within the given nominal level. However, the Type I error rates of the PB test is closer to the intended level than the GV test. Overall, in terms of Type I error rate and power, the PB test performs well regardless of the sample sizes, values of the variance components, dimensions of regression coefficients. Therefore, one may recommend to use the PB test as a rational alternative in practical because of its effectiveness and simplicity.

5. Examples

Example 1. The dataset was considered in Munnell (1990) and Baltagi and Pinnoi (1995), they considered the Cobb-Douglas production function relationship investigating the productivity of public capital in private production and estimated the variance components and regression coefficients. Swamy and Arora (SWAR) estimations of component variances are used, they are set as $\hat{\sigma}_\varepsilon^2 = 0.034$, $\hat{\sigma}_\mu^2 = 0.083$, $\hat{\sigma}_\nu^2 = 0.010$. The estimates are the best quadratic unbiased (BQU) estimates as defined in (2.5). The number of groups is 48, the observations of per group are 17. In this example, the homogeneous linear hypothesis $H\beta = 0$ is interested, which is a special case of the hypothesis in (3.1), the p-values of both tests are computed using Monte Carlo simulation consisting of 1,000,000 runs. Using algorithms for the proposed PB approach, the PB p-value is obtained as 0.0640. On the other hand, the p-value of the GV test under the same estimates of variances is computed as 0.1006. Despite the p-values of the PB and GV tests are greater than 0.05, the proposed PB approach performs better than that of the GV test.

Example 2. The datas are derived from World Statistics Pocketbook, International Statistical Yearbook, China Statistical Yearbook for Economy, China Trade And External Economic Statistical Yearbook. They are 1995-2002 editions, the datas in 1998-2002 are used in our simulations. The following model was considered.

$$\ln Y_{i,t} = \beta_0 + \beta_1 \ln \mathrm{F1}_{i,t} + \beta_2 \ln \mathrm{GDP}_{i,t} + \mu_i + \nu_t + \varepsilon_{i,t}, \quad i = 1, 2, \cdots, 6;\ t = 1, 2, \cdots, 5.$$

where, for state i in year t, the regions included are Hong Kong, Taiwan, Japan, Korea, EU and the United States, $Y_{i,t}$ denotes the amount of China's export, $F1_{i,t}$ is the amount of Foreign Direct Investments, $\mathrm{GDP}_{i,t}$ is the amount of gross domestic product in these regions. Under the hypothesis $H_0 : H\beta = 0$, the p-value of the proposed PB test is estimated as 0.0400, the GV p-value is computed as 0.0626. Therefore, at significance level 0.05, the proposed PB and GV tests leads to the opposite conclusions.

6. Concluding remarks

The available approaches for testing regression coefficients in the two-way error component regression model have serious Type I error problems when the sample is small and moderate. In this paper, a simple and yet efficient procedure is suggested, namely, the parametric bootstrap approach. Moreover, we study the properties of the proposed test for small, moderate and large samples by Monte Carlo simulation including the behavior of the Type I error rates and powers, respectively. The properties of the generalized variable approach in [4] and the proposed test are compared. As the numerical results indicate, the PB test has satisfying Type I error control for all the sample sizes while the generalized variable test tends to be unreliable as the sample sizes are small, the powers of the PB test is a little higher than the GV test. Overall, in terms of Type I error rate and power, the PB test consistently performs well, regardless of the sample sizes, values of the variance components, dimensions of regression coefficients. We would like to stress that attention should be taken regarding the choice of the test, because the different tests can lead to different conclusions, in terms of accepting or rejecting the null hypothesis.

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